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RISK-INFORMED STRUCTURAL DESIGN OF THE FUTURE

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Abstract: *We present a conceptual framework, and a computer tool, to achieve a more rational, risk-informed, and hopefully optimal design of structures. Within this framework, we present a novel, experimental approach towards structural design, which is able to:*

- *Formalize the design process via algorithmic means*
- *Perform fast time-domain, nonlinear structural analysis of many design alternatives in a short time span*
- *Evaluate the consequences of each design alternative both in terms of initial cost as well as in terms of future costs due to earthquakes*
- *Provide the designer with means to rank design alternatives and choose the best*

Following traditional risk-evaluation theory, the implementation shown consists of four independent modules: Design, Hazard, Structural Analysis, and Loss Calculation.

The Design Module takes as input a building specification and generates a detailed and realistic analytical structural model alongside a complete description of the assets contained therein. The Hazard Module allows the user to specify hazards by way of providing an annual rate of exceedance of pseudo-acceleration curves and a set of representative ground motion records. The Structural Analysis Module runs a set of Incremental Dynamic Analyses (IDA) using the predefined records and hazard levels on the designed building. This result set contains the complete time-history response of all degrees of freedom for any desired engineering demand parameter (EDP) e.g. peak drifts, reactions, floor accelerations, etc. Finally, the Loss module uses this information to estimate the distribution of losses for the assets of the structure. The output of the module is a set of loss measures, such as the annual rate of exceedance of loss and the expected annual loss, among others, along with the construction costs.

While this is a standard risk-evaluation procedure, the implementation presented allows for very fast design, analysis, and risk computation of many design alternatives by automating and abstracting away the difficult and computationally expensive processes such as structural design, analysis, and asset loss estimation. This allows one to effectively explore the space of structural designs to eventually find global optima for a given risk measure, or at least better solutions than those associated with coded design.

We also envision that, as our society becomes more risk-averse and computational power cheapens, Building Codes will tend to increasingly adopt a version of criteria based on monetary —or other types of— risk and initial building cost as ingredients of its measure of good performance.

1. Introduction

Buildings in seismic zones might suffer severe damage and even collapse during high-intensity events, which cause human and monetary losses that impact negatively on society. For this reason, building codes enforce strict rules on designers, aimed at fulfilling limit states of life-safety and collapse-prevention for their corresponding medium and high return periods respectively. However, they consider monetary and human losses only as indirect proxies to good performance, and therefore it is unclear whether these losses are tolerable by the owners or by society. This is unsatisfactory.

Furthermore, risk analysis is rarely carried out in actual structural engineering projects. This is due, in part, to the absence of good tools to do so in a practical manner, because it is a data and computation-intensive task, particularly for tall buildings or for multiple design proposals. To achieve this, it is necessary to evaluate the behavior of the building against events whose intensity is smaller, close to, and above the design capacity, using sophisticated models that are necessarily nonlinear. This procedure has to be repeated for multiple design options which, at the moment, is not feasible.

The authors are unaware of a procedure that allows the analyst to explore the design space effectively searching for global optimum for a given risk measure, or at least better solutions than those associated with codified design. If these tools existed, we would be in a position to understand the effect of structural design on risk, and we could start thinking seriously about moving toward risk-informed design procedures.

Objective

This paper has the following objectives:

- To develop a framework that allows the designer to achieve a more rational, risk-informed, and hopefully, optimal design of structures.
- To present a computer tool alongside this framework, which among other things, allows for very fast structural design, analysis, and risk computation of many design alternatives, by way of automating and abstracting away the difficult and computationally expensive processes.
- To use the presented framework and computer tool to study the influence of the design procedure on its seismic risk.
- To present a case study analyzing and comparing the level of risk achieved by two different design procedures for the same building specification.

Scope

This study is part of the first author’s doctoral thesis. Therefore it is so far restricted to:

- Planar framed reinforced concrete buildings designed with the 2017 version of the Mexican code
- An occupation class and assets of public offices commonly found in Mexico City.
- A hazard scenario that corresponds to the design period of the buildings and seismicity of Mexico City alongside representative strong motion records recorded in soft soils
- Estimation of monetary losses only, which means that no downtime or human losses are considered

2. Optimum design of buildings

Optimum design has been a primary goal of earthquake engineers since the inception of the field; however, it has been relatively unexplored. The first studies on optimum design by Esteva (1967), Rosenblueth (1976a), Whitman and Cornell (1976) made the following hypotheses:

1) that occurrence in time of future earthquakes follows a Poissonian process;
2) that the initial cost of the building and future losses depended on a single intensity measure, *c*, the base shear demand;
3) that the vulnerability model is of complete fragility, that is, when the demand *c* exceeds the capacity *c_s* then total loss takes place;
4) that the building is restituted immediately to exactly the same conditions in strength and stiffness as it was before it suffered damage.

Even for the simplified approach in which a building is characterized by a single design parameter*,* it was found that the optimum return period changes depending on the hazard and that this optimum always produces return periods that are shorter for high-seismicity zones and longer for low-seismicity zones (Esteva 1967, Rosenblueth 1976b, Ordaz et al. 2017). The main insight derived from this observation was that it is wise to buy lateral strength when it is cheap and to avoid buying too much strength when it is expensive. This means that in areas of low seismicity, one should design for very long return periods, and conversely, one should design for short periods in areas of high seismicity. This interesting result implies that designing structures by specifying a constant return period for all zones is not optimum.

Intuitively, the cost function associated with a design decision parameter c_s is the sum of the initial cost of the building today, C_I, plus the present value of the future losses due to earthquakes, C_{FL}, and so the *expected present value cost* associated with a design decision is (Ordaz et al. 2017)

$$C(c_s) = C_I(c_s) + C_{FL}(c_s) = C_I(c_s) + (1 + S_L)\mathrm{AAL}/\gamma \qquad (1)$$

Variable c_s might be interpreted as a measure of the lateral strength, such as a seismic design coefficient, or more broadly, as a particular building design instance. In this equation, γ is the discount rate which accounts for the value of money in the future. This factor can also be interpreted as our measure of the importance of failures that would happen far in the future. If γ is very low, this implies that we care more about events in the distant future than in the near future. The numerical values of γ have been relatively constant across the last 200 years and range from 0.03 to around 0.05 (Picketty, 2013).

The term $(1 + S_L)\mathrm{AAL}/\gamma$ is exactly the expected present value of future losses regardless of the vulnerability model (Ordaz et al. 2017, Appendix). In this equation, $\mathrm{AAL} = E(L)\nu_0$ is the average annual loss due to earthquake perils, where $E(L)$ is the expected loss for the next random earthquake, and ν_0 is the expected number of earthquakes per unit time.

This equation shows that future losses are proportional to the AAL, in fact, $1/\gamma$ times as many years of average annual losses. One could have naively thought to see the product of AAL times the life span of the building, e.g. 50 times, however, this is in fact not the case, as future money is usually worth less than what it is worth today. For the standard values of γ of 0.03 or 0.05 this amounts to 33.3 and 20 years respectively, which is less than the usual lifecycle of a building which is around 50 years.

The secondary losses are represented by S_L (as fractions of C_{FL}), which can be caused by the complete or partial collapse of the building, and account for the fact that there might be other factors to protect besides building cost. The numerical value of S_L ranges from 0, in the case where there is no collateral damage, and up to 12 or more, in the case where collapse, among other things, causes severe collateral damage. In other formulations, S_L could account for the value of lives lost as a consequence of building damage and collapse. This leads naturally to the question of the equivalent monetary value for a human life which is a difficult topic (Salgado et al 2016) that we will avoid in the present work.

According to game theory, a perfectly rational decision maker would minimize the expected value of the cost function (Von Neumann & Morgenstern 1953) and not other statistical properties; therefore, optimum design means minimizing the expected value of Eq. 1 by finding a suitable building design instance. To design a building optimally in this context means that for a fixed (γ, S_L) pair, one has to find the building c_s, such that C in Eq. 1 is minimum. C is again, the expected cost of the decision to undertake the construction of a particular design instance today.

Taking this reasoning to its logical conclusion implies that building codes could be drafted to include a single instruction, which would be to demand that the proposed designed building minimizes Eq. 1 (Rosenblueth, 1980). This raises some interesting questions: are there additional restrictions we must place upon design besides optimizing present cost? What if collapses and human losses are too frequent? Perhaps a minimum lateral strength should be specified or alternatively a minimum return period. Perhaps if the risk underlying the decisions were more easily known, society could have a more direct say in what level of risk is acceptable. And that is the purpose of this paper.

3. Description of the conceptual framework and program

To achieve the objective of this research, the following steps are required

1. Formalize structural design via algorithmic means
2. Automate the placement of assets according to an occupancy class
3. Specify the corresponding seismic hazard and representative ground motion records
4. Run IDA analysis on the designed building
5. Compute losses and risk measures using the results from the IDA and the hazard and assets

Following traditional risk-evaluation theory, and identifying the independent statements of the aforementioned steps, the implementation developed consists of four independent modules, each with a single responsibility (Fig. 1): Design, Hazard, Structural Analysis, and Loss Calculation. Each of these modules attempts to abstract away the complex processes, such as structural design or analysis, and minimize the possible human errors;

- The goal of the design module is to take in a minimal building specification and produce a complete finite element model, along with a detailed description of the building assets.
- The goal of the hazard module is to generate a hazard object that contains the annual rate of exceedance of intensity and to store its corresponding representative records.
- The goal of the structural analysis module is to perform a fast time-domain, nonlinear structural analysis of many design alternatives under many strong-motion recordings in a short time span, producing an exhaustive set of results for all degrees of freedom.
- The goal of the loss module is to evaluate the consequences of each design alternative both in terms of initial cost as well as in terms of future costs due to earthquakes. The program must therefore provide the designer with a means to rank design alternatives and choose the best

The conceptual framework aims to achieve a more rational, risk-informed, and hopefully optimal design of structures.

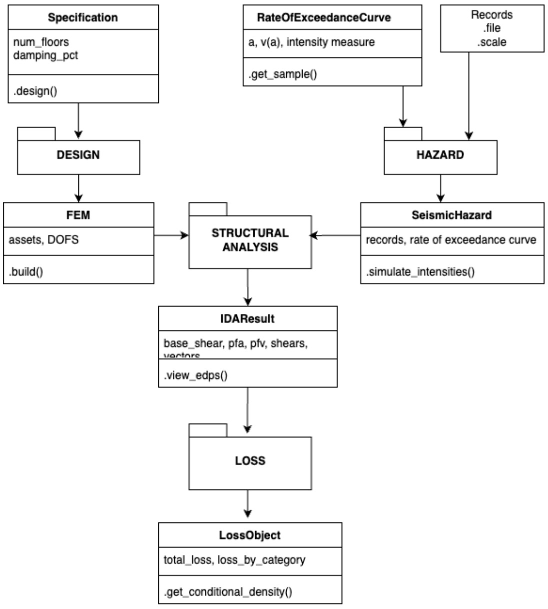

Figure 1. Flow chart for the program, data flows from top to bottom. Each module is specified by a folder icon and each object by a rectangle. The output is a loss object containing the conditional probability density of losses given an intensity measure, as well as a full breakdown of all the individual asset losses.

4. Automated structural design module

This module must produce a finite element model capable of being implemented and executed by a nonlinear structural analysis software. Furthermore, it must produce a complete description of the assets of a building, information that is required to perform a loss computation after the structural analysis results are obtained.

A designed building is the realization of a process that starts with a specification. In turn, a design process is an algorithm that outputs the stiffness and strengths, no matter how complicated, of the structural elements, as well as the parameters required to implement a model into a structural analysis program. For example, one needs to specify masses and critical damping percentages in order to perform linear response history analyses, while member strengths are required to perform pushover analysis. The novel approach presented in this work is to implement this procedure incrementally and iteratively (Fig. 1), starting perhaps with a simple rule that assigns masses according to some codified procedure, and then assign stiffnesses to achieve realistic natural vibration periods and shapes. The procedure continues in this manner, refining the model until it achieves the properties we are looking for in the final building, such as a desired level of strength and ductility.

However, the aforementioned description of the design process is incomplete because a finished building also contains assets. Therefore, we must broaden our definition to include the automatic generation of a complete description of its assets. This information is required for risk and loss estimation. The definition and location of the assets are obtained by establishing which occupancy class this building belongs to. For example, a hospital holds different assets compared to an office. It is also noted that given the same specification, two engineers will almost surely produce different solutions or "designs", either because they used different methods or have different criteria as to what constitutes an acceptable result.

Therefore, it is evident that a clear distinction between a specification and a designed instance exists and as such must be made in a computer program. It is evident as well, that a specification with different occupancy classes and design criteria will produce different building instances and therefore incur different losses for the same seismic events. Thus, a building specification is a high-level container of information needed to produce a design instance. The authors have found that by reducing the specification to a given building archetype, a shorter and simplified list of parameters is obtained. This allows for a declarative way to talk about buildings of a given archetype.

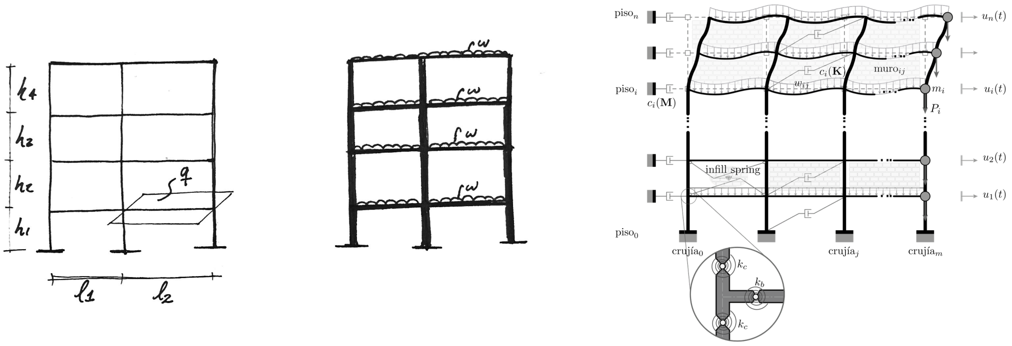

Figure 2. Iterative design process, from specification to codified mass assignment, to stiffness assignment, and finally to strength assignment and a refinement of the model ultimately expressed as a finite element program.

5. Hazard module

Seismic hazard expresses the relationship between a measure of seismic excitation and its annual rate of exceedance at a specific location. This relationship is usually given by a hazard curve, of a chosen intensity measure, such as first-mode spectral pseudo-acceleration. Risk-based assessment needs to bind structural responses to rates of exceedance using one or more intensity measures such as spectral displacements or pseudo-accelerations. The goal of intensity-based scaling methods is to provide scaling factors for records so that nonlinear analyses of the structure for these scaled records give an accurate estimate of the probability distribution of the engineering demand parameters (EDPs). We require that this procedure is also efficient, i.e., it minimizes the record-to-record variability in the structural response (Kalkan 2010). The responsibility of this module is to generate an object that represents the seismic hazard and to store its representative ground motion records. The user can specify the rate of exceedance curve, using a fixed number of points or via an analytic expression such as a Pareto-type curve. This module is decoupled from the rest of the program, which allows the user to generate sets of "hazards" to be used in comparative or parametric studies. In general, the hazard module would contain an exceedance rate curve of the selected intensity measure plus a set of accelerograms equally likely to be considered representative of the ground motion associated with a certain exceedance rate.

6. Structural design module

This module uses the records defined in the previous module to run Incremental Dynamic Analyses (IDA) (Vamvatsikos and Cornell 2002, Vamvatsikos 2004) for a designed building, producing an exhaustive set of results that contain, among other things, the response for all degrees of freedom for any demand parameter such as reactions, accelerations, velocities, displacements, etc.

The fundamental goal of an IDA is to capture the variability of the structural response against a set of records with distinct frequency contents and increasing intensity, in order to gain insight into the dynamical behavior of a structure. To do this, one employs a suite of selected ground motions. For each ground motion, the structure is subjected to a monotonically increasing intensity (in pseudo-acceleration), obtaining, for each record and each intensity, the structural response in terms of EDPs, e.g. peak inter-story drift. In this manner it is conjectured that the response of the building across these intensity levels and with distinct frequency content could offer insight into its dynamical capacity and performance, starting from the elastic stage up until the onset of collapse.

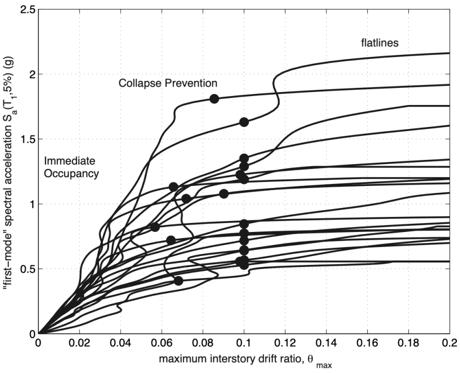

Figure 3. IDA curves for a 15-story steel building (Vamvatsikos and Cornell 2004), black lines correspond to a record/ intensity combination, black dots correspond to the onset of collapse e.g. collapse prevention damage state defined as 20% of the elastic stiffness.

The set of IDA curves plotted in EDP vs IM fashion $(\theta, S_a(5\%, T_1))$ is called an IDA-plot (Fig. 3). Using these results, one can statistically summarize the structural response as a conditional probability distribution of the EDP given an intensity. Moreover, one can estimate the probability of collapse given a level of intensity, which is a useful quantity in damage analysis and risk analysis in general. For example, one can compute the median intensity at which collapse caused by dynamic instability is achieved. In this procedure, collapse can be identified as a softening in each curve that can be identified visually as a flat line. This softening corresponds to numerical instability, and if the model satisfies some basic robustness properties, then this corresponds to dynamical instability (López 2016).

7. Loss Module

The responsibility of this module is to estimate the conditional probability of loss given an intensity, for all the assets of a building. For this purpose, it uses both the results from the IDA, as well as the hazard curve, and the specification of the assets generated by the design module. Assets can be classified into three categories; structural, non-structural, and contents. The fundamental difference between the non-structural elements and contents is that the former refers to fixed elements, while the latter is usually mobile and is typically inventoried.

In the literature, human health and business uptime are sometimes also considered assets. In general, they require separate treatment and will not be considered as such nor discussed further herein. Henceforth, all assets are physical objects located somewhere in or on the outside of a building.

Figure 4. Examples of assets from the three different categories; a) structural elements (beams, columns and walls), b) non-structural elements such as marble tiling, c) contents (plotters, printers, chairs, tables).

Another useful classification pertains to the way that an asset responds to actions. This means paying attention to where loss can come from during events.

In this spirit, we say an asset is sensitive to a demand parameter such as acceleration, displacement (drift), force, or velocity when this is the primary cause of loss. For example, a flower vase (Fig 5a.) is acceleration sensitive, while most lateral force resisting structural elements (Fig 5b.) are drift sensitive. Conversely, we say an asset is rugged (Fig 5c.) when it is unaffected by any local action, and therefore the only cause for monetary loss would be total or partial building collapse. The terminology comes from a rug which is the representative element of this class. Other examples of this class would be sofas and clothing or shoes.

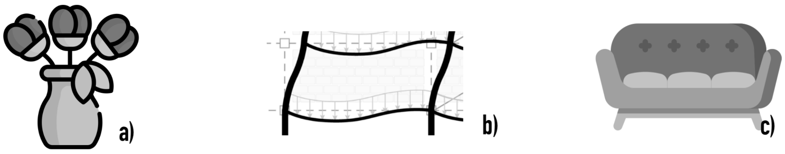

Figure 5. Example representation of; a) acceleration sensitive asset (flower vase), b) drift sensitive asset (beams and columns), c) rugged asset (sofa).

The main contribution of this module is to show how to compute the direct loss for structural elements based on physical principles. We propose a method for reinforced concrete beams and columns based on a modification of the energy dissipation capacity index proposed by Park and Ang (1987). We postulate that any member possesses a reference inherent hysteretic energy dissipation capacity that is independent of loading history and that the piece is able to dissipate a certain amount of half cycles worth of energy before it collapses from degradation. It has been observed (Park and Ang 1985, Kunnath 1997) that while cyclic degradation is important, a more sound approach is to consider a linear combination of cyclic capacity and monotonic ductility. This index ranges from 0 to 1, and consists of two terms; the first term is related to the ratio of the maximum ductility demand to the ductile capacity achieved by the section during the loading cycles.

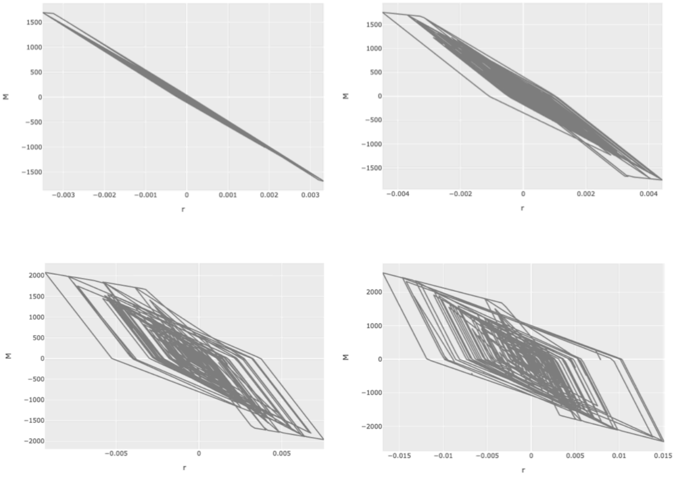

Figure 6. Damage progression in a RC column joint (chord rotation vs moment), loss is computed directly from the physical hysteresis observed, that is, a combination of the energy dissipated during the loading cycles and the ductility demand.

Therefore, the repair costs estimated by this module are proportional to the damage index value computed for each structural element and subelement for a given run. The scenario of total building collapse is taken into account with the criterion expounded in the IDAs section, in which case all assets will incur in total loss.

Regardless of the procedures used, the loss measure we are most interested in is the annual-rate-of exceedance-of-loss curve. This curve tells us how frequent a given level of loss is exceeded, and contains all the information to compute other loss measures, such as the *AAL* or the expected loss.

This curve is computed in the following manner; fix a level of loss, say *L*, then, for all records and all intensities, sum the corresponding frequency of events that caused losses greater or equal to *L*; this is the rate of exceedance of that level of loss. Repeat this procedure for all loss levels from 0 to the total building cost.

We are also very interested in *AAL*, since it is necessary to minimize Eq. 1. This measure can be computed as the product of the expected loss for each event of the set and its corresponding annual frequency.

8. Case Study

We study the influence of the design procedure on the cost function (Eq. 1) of a public office building designed using two different force reduction coefficients from the 2017 Mexican code. This code uses the modal spectral design method, using force reduction factor *Q*, which reduces the design lateral force given by $V = c_S W / RQ$.

With this in mind, we have selected as case studies the values Q=1 and Q=4, holding everything else constant. These designs are henceforth called Q1 and Q4, correspondingly. Intuitively, Q1 is a stronger design, while Q4 should have more ductile capacity at the cost of some strength. The seismic hazard for the buildings planned to be built in the soft-soil lake zone (III B) and with a natural period of 0.5 s was computed with CRISIS (Ordaz et al 2021), see Fig. 7.

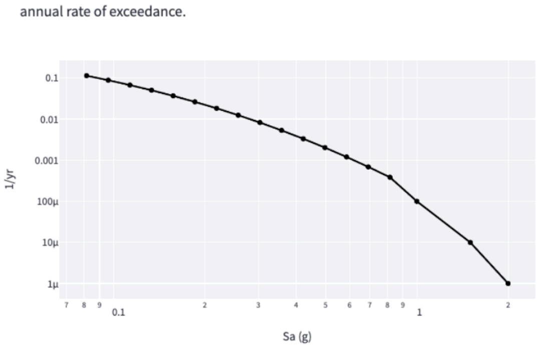

Figure 7. Rate of exceedance curve (in log-log) used for the sample buildings, which is representative of the soft lakebed soil zone in Mexico City.

A set of 50 representative strong motion records tied to the presented seismic hazard were selected. The geometrical configuration of the building consists of 3 stories of equal heights of 3.5m and widths of 6, 6, and 8m, respectively. The fundamental period of this building is 0.48 s.

The distribution of assets follows empirical data of public offices commonly found in Mexico City (Reynoso 2013).

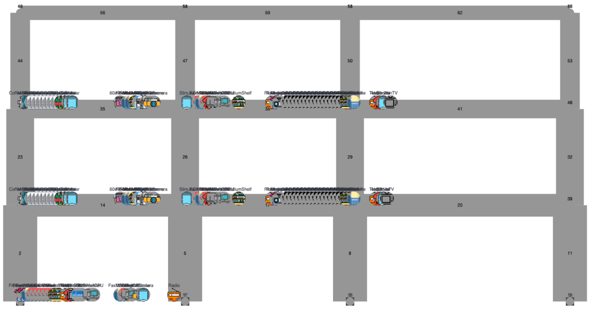

Figure 8. Geometrical configuration of the framed RC building, with a schematic representation of its assets.

Cost distribution

Since there is only a change in the strength of the building between Q1 and Q4, nonstructural and content costs are numerically equal, whereas Q1 costs around 10% more than Q4 in total, primarily due to an increase in longitudinal steel required to achieve the desired level of strength. Structural cost increased from $51k USD to $73k USD, going from Q4 to Q1, which is a 43% relative increment. Figure 9 shows the distribution of costs as functions of the asset categories.

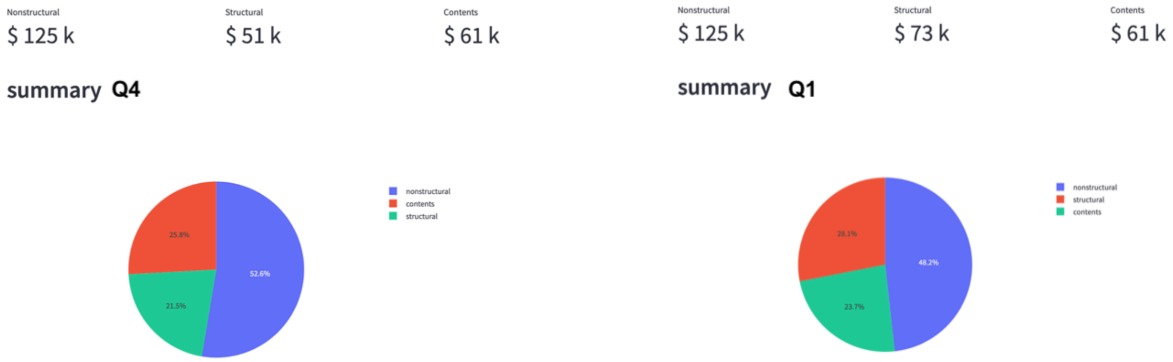

Figure 9. Asset cost distribution (in categories, blue-nonstructural red-contents green-structural) for Q1 and Q4, The total cost of Q1 is $237 thousand dollars, and Q4 is $259k. While nonstructural and contents remain the same, Q4 costs about $22k less than Q1.

Pushover curves

The first mode pushover of both buildings is shown in Fig. 10. Interestingly, even though Q1 was designed with Q=1, which implicitly measures the theoretical ductility of the building, the actual ductility developed in a static pushover analysis is around 2, while for Q=4, the ductile capacity was around 8. This indicates that in general, it seems hard to build a real structure with no reserve ductility using a codified design. This is a consequence of the fact that most non-trivial structures contain some reserve strength and ductility that are not present in in single-degree-of-freedom systems.

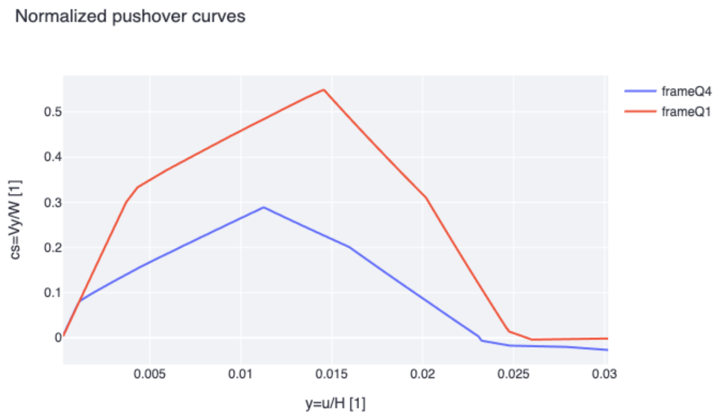

Figure 10. Normalized first-mode pushover curves for Q1 (blue) and Q4 (red). The ordinates are normalized with respect to the total weight (and are therefore numerically equal to the seismic coefficient) and the abscissas are normalized with respect to the height i.e. total drift.

Mean IDA curves comparison

An IDA was performed with the 50 selected records at increasing intensities. Figure 11 shows the median normalized IDA curves for both buildings. We can observe that they are relatively similar; each of them is normalized with respect to their yield intensities and static drift yields. The similarity in both curves highlights the fact that only decreasing the strength and/or increasing ductility for a fixed building does not intrinsically change its dynamical behavior.

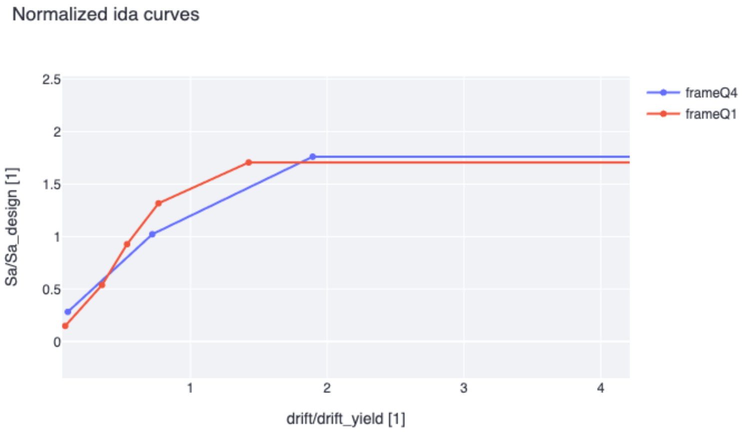

Figure 11. Normalized mean IDA curves for Q1(red) and Q4 (blue). The ordinates are normalized with respect to the Sa of design while the abscissas are normalized to their corresponding drift at yield computed from the static pushover.

Rate of exceedance of loss comparison

We computed the annual rate of exceedance of loss for both buildings, see Fig. 12; we can observe that Q1 is always below Q4, which means that on average, for a fixed loss, Q1 suffers that loss less frequently than Q4; this makes sense as Q1 has more strength.

Seeing those results, it would intuitively seem that Q1 is a better design than Q4, as the latter does not seem to show any advantages so far. It is worth noting however, that the AAL of Q1 is 0.5k USD, while the AAL of Q4 is 0.9k USD, this means that although Q1 is more expensive to build, it suffers less future losses than Q4, which is cheaper to build today. This agrees with experience.

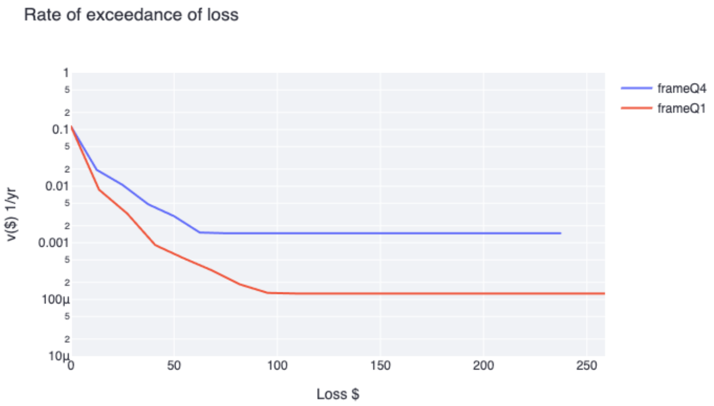

Figure 12. Annual rate of exceedance of loss curves for Q1 (red) and Q4 (blue). Note that for all levels of loss, Q1 is below Q4, this implies that it suffers the same loss less frequently.

Expected present day cost comparison

Finally, we present a comparison of the expected present-day cost of constructing Q1 and Q4 a function of γ, the monetary discount factor (Fig. 13). We observe that, contrary to intuition, the curves cross, which means that our initial naive idea about Q1 being always the better design (reasoning from the loss exceedance curve) is perhaps not correct.

Using the interpretation of γ, the results shown allow us to infer something very interesting: if γ is low, this means that future losses really matter and so Q1 is clearly a better design. Conversely, when γ is high then we care little about future losses, in which case the design with the lowest initial cost tends to win; thus, Q4 would be a better design choice.

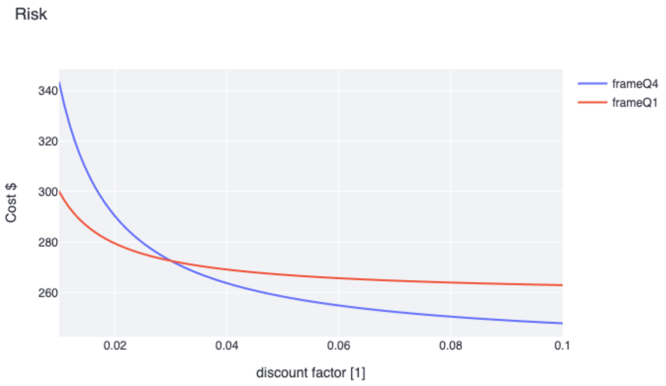

Figure 13. Expected present day cost of constructing Q1 (red) and Q4 (blue) as functions of γ, the monetary discount factor. Notice that the curves cross at around 0.03, in which both design options yield the same present value cost.

9. Conclusions and observations

The bases for a conceptual framework that allows the designer to achieve a more rational, risk-informed, and hopefully optimal design of structures were presented. This framework was implemented in a program that allows the engineer to perform structural design and analysis in quick succession, and to choose the best among many building design options. We believe this tool is useful for both research and practice as it allows a quick comparison of many design alternatives by automating and abstracting away the difficult and computationally expensive processes such as structural design, analysis, and asset loss estimation.

Using this implementation, the influence of the design procedure on the structural risk was studied, where the expected present cost of undertaking (Eq. 1) was shown as a function of γ the monetary discount factor. This comparison was performed between two buildings, where the selection of a force reduction factor Q was shown to directly influence the initial costs and future losses suffered by the building.

The main finding of this research so far is that for the selection of the optimum building, it is not enough to look at measures of behavior and structural performance such as pushover or IDA curves, nor to base our criterion on the rate of exceedance of loss curve, or any other loss measures on their own. Rather, Eq.1 must be analyzed for the building options at hand. The case presented showed that the optimum design changed from a more resistant design with lower Q (force reduction factor) when γ was low, to a less resistant and more ductile design (with higher Q) as γ increased.

Preliminary results from this investigation suggest that an adequate estimation of the monetary discount factor γ is crucial in choosing the optimum design. Future work in this regard intends to study the consequences of a wider variety of building archetypes, construction materials, and design criteria on the optimum design, while revising the notion that simple rules are able to effectively lead us to tolerable losses.

9. References

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