

TOWARDS MORE EFFICIENT FOOTINGS FOR CONCENTRICALLY BRACED FRAMES: EFFECTS OF VARIABILITY

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Abstract: *Footings for steel concentrically braced frames are a major contributor to the overall cost of the seismic force-resisting system (SFRS). In general, there are two approaches to design a footing for earthquake loads. Approach 1 involves designing the footing to resist the capacity of the SFRS. Alternatively, Approach 2 entails designing the footing to withstand the design seismic load, which has been reduced from the elastic earthquake demand in accordance with the ductility of the SFRS. The second approach can produce much smaller footings than the first approach, but considering the overstrength of the SFRS, it is likely that the real seismic demands on the footing and underlying soil will exceed the design demands. As such, the size of the footing is an influential factor in the response of the buildings. Additionally, to reliably predict the seismic behaviour of buildings, it is important to consider the inherent uncertainties in the system. Consequently, it is necessary to investigate how different footing sizes affect the building's performance while taking into account both system uncertainty and record-to-record variability.*

In this study, a 2-storey concentrically braced frame (CBF) building with an X-bracing configuration is selected to study the effects of footing size on the behaviour of the building. This building is located in Vancouver, Canada, on a site Class D condition. The superstructure is designed following the requirements of the Canadian design code and standards, while the footing size is bounded between the most conservative Canadian approach (Approach 1 above) and the least conservative American approach (Approach 2). An advanced computational model, including gravity framing, is developed in OpenSees, and the seismic performance is assessed through nonlinear response history analysis. The uncertainty of the system parameters, including the properties of the superstructure and substructure, as well as uncertainty in the seismic demand on the building, is accounted for using Latin hypercube sampling. The findings of this research suggest the potential for finding an efficient footing size for Canadian low-rise CBF buildings that achieves desirable seismic performance without undue construction cost.

1. Introduction

Concentrically braced frames (CBFs) are widely recognized as efficient and cost-effective systems for withstanding seismic lateral loads. While the capacity design philosophy is widely employed to prevent

nonlinear response in frame members other than braces, the footings of these frames are treated differently in various design codes. Generally, there are two approaches for the seismic design of the foundations. In the first approach, the footings are designed for the capacity of the lateral seismic force resisting system (SFRS), whereas in the second approach, the footings are designed for lateral earthquake load derived from the reduction of elastic earthquake demand based on the ductility of the SFRS. The first approach produces footings that are larger than when the second approach is used. Therefore, due to the system overstrength, it is possible that the demand on the footing surpasses the seismic demand used in the second approach. Several studies (Koboevic and Murugananthan 2019a; b; Madani *et al.* 2022; Wichman *et al.* 2022) showed the significance of considering the footing and the soil underneath on the overall seismic behaviour of CBF buildings. Moreover, they showed that the size of the footings affects the overall performance of the buildings. For instance, Koboevic and Murugananthan (2019b) studied 3-storey CBF buildings with Canadian-designed footings located in Vancouver and Montreal on both soft and stiff soil. They found that buildings with not capacity-protected (NCP) foundations experience larger drifts compared to buildings with capacity-protected (CP) foundations.

The uncertainty in material properties, gravity loads, and seismic demand, in addition to footing size, have a meaningful impact on the performance of CBF buildings. Given the variability in soil properties and the challenging nature of accurately determining characteristics such as soil stiffness and strength, it is crucial to consider the uncertainty related to these properties in response history analysis when taking into account the soil-foundation-structure interaction (SFSI) (Bazzurro and Cornell 2004; Moghaddasi *et al.* 2011; Raychowdhury 2009). Madani *et al.* (2023) investigated the effects of variability in soil properties on 1- and 2-storey CBF buildings with NCP foundations under design-level earthquakes. Their findings indicated that variations in soil properties have the potential to not only alter drift values but also change the primary contributor to drift. Those findings were drawn from independently changing one parameter and observing its impact on the inter-storey drift of short-period CBF buildings. However, the combined impacts of simultaneous variation in all random variables on the behaviour of CBF buildings were not investigated.

The objective of this study is to examine the possibility of designing more efficient foundations for Canadian CBF buildings. To achieve this, a 2-storey CBF building located in Vancouver, Canada, on Class D ($180\text{m/s} \leq V_{s30} \leq 360\text{m/s}$) soil with X-bracing as the lateral force-resisting system is evaluated. This building is designed per the National Building Code (NBC 2020) and steel design standard (CSA S16:19 (2019a)). The numerical model is developed in OpenSees (McKenna *et al.* 2000). The gusset plate connections and shear tab connections are explicitly modelled, as are gravity framing and brace fracture. A Beam on Nonlinear Winkler Foundation model is used to consider the impacts of soil and foundation on the performance of the structures. In order to address the uncertainties arising from random parameters in the structural system, an efficient incremental record-wise Latin Hypercube Sampling (LHS) (Vamvatsikos 2014) is employed to propagate the uncertainties from multiple parameters to determine the demand and capacity of the system. This approach ensures a comprehensive consideration of the collective impact of these variables. In this study, the size of the footing is treated as a decision variable, and the impacts of its variation on engineering demand parameters such as drift and acceleration are examined.

2. Building Design

This study focuses on evaluating the seismic performance of 2-storey office buildings located in Vancouver (49.261° N , 123.114° W), Canada. This building has a seismic hazard corresponding to seismic category *SC4* with a class Site D ($180\text{m/s} \leq V_{s30} \leq 360\text{m/s}$) soil condition. The lateral force-resisting system employed in the east-west (E-W) loading direction consists of moderately ductile tension-compression concentrically braced frames (CBFs) with a ductility reduction factor (R_d) of 3 and an overstrength factor (R_o) of 1.3. The members of the superstructure, including braces, beams, and columns of both braced and gravity frames, were designed in accordance with NBC 2020 and CSA S16:19. The short-period building of this study had a base shear coefficient of 0.3. In the calculation of seismic design forces, the equivalent lateral force procedure was employed. Figure 1 shows the plan view and elevation view of the 2-storey archetype building studied here.

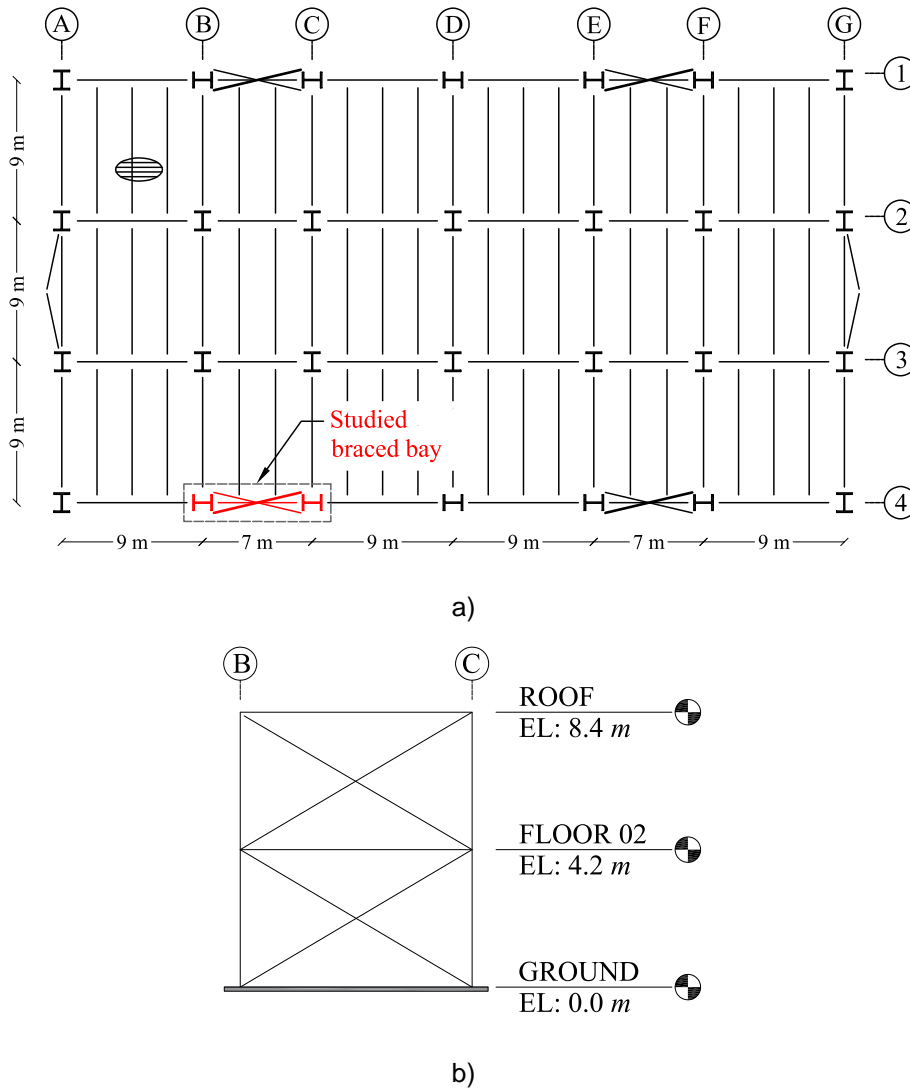


Figure 1. Studied 2-storey CBF building: (a) plan view; (b) elevation view

Square hollow structural sections (HSSs) were selected for the braces, whereas wide flange sections were selected for the beams and columns. The brace members satisfied the local and global slenderness ratio requirements of CSA S16:19. In order to allow for brace end rotation, the gusset plate connections were designed with a linear offset of $2t_p$, where t_p is the thickness of the gusset plate. The capacity design requirements of the steel design standard were followed to select the beams and columns of braced frames. A deterministic value was used for gravity loads and material properties to design the superstructure. Table 1 lists the sections selected for the braces, beams, and columns of the braced frame.

Table 1. Section sizes of CBFs

Building	Storey	Braces	Beams	Columns
2-storey	2	HSS 89 × 89 × 4.8	W410 × 39	W310 × 79
	1	HSS 102 × 102 × 7.9	W410 × 60	W310 × 79

This study examined two common soil types, referred to here as stiff and soft soil. In order to design the footings of buildings, a deterministic value was selected for all soil properties. The soft soil had a shear wave velocity (V_{s30}) of 183 m/s to be representative of the boundary between site classes D and E, while stiff soil was indicative of site classes C and D boundary with a shear wave velocity (V_{s30}) of 354 m/s. The size of the footing was a decision variable in this study, and it could not be smaller than the footing designed per ASCE 7-22 (2022) and ACI 318-19 (2019) (referred to as US footing) nor larger than a Canadian capacity-protected

footing designed per NBC 2020 and A23.3-19 (2019b) (referred to as CP footing). US footings are designed for factored gravity loads and the code-specified lateral forces. CP footings are required to withstand factored gravity loads and the expected capacity of the SFRS system, except that it is not necessary for the designed moment to exceed the overturning moment associated with $R_d R_o = 1.0$. The sizes of the US, CP, and NCP footings designed for the 2-storey archetype building are listed in Table 2.

Table 2. Dimensions of the footings of the 2-storey archetype building

Type of soil	Footing dimensions (m) (Length*Width*Depth)		
	US footing	NCP footing	CP footing
Stiff soil	12.0*1.9*0.6	13.8*3.8*0.9	14.7*4.5*1.0
Soft soil	13.4*2.7*0.6	14.7*3.4*0.8	15.5*3.7*0.9

3. Numerical Modelling

The 2-storey building was modelled in two dimensions using OpenSees (McKenna et al. 2000). The seismic weight and gravity loads were computed in accordance with NBC 2020. To incorporate P-Delta effects, the braced frame was connected to a frame representative of the interior gravity framing system of the building. The gravity framing was explicitly modelled as proposed by Elkady and Lignos (2015). The corotational transformation method was employed within the OpenSees framework to incorporate the second-order (P-Delta) effects. Mass and stiffness proportional Rayleigh damping was employed to account for inherent damping. In this study, the damping ratio was taken as a random variable, which was assigned to the two first modes. The damping ratio was assumed to have a triangular distribution with 0.5%, 2%, and 3.5% as the lower limit, peak location, and upper limit, respectively. A summary of the modelling assumptions is presented in Figure 2.

The modelling of the CBF system braces followed the guidelines suggested by Sen et al. (2019). This involved employing 16 nonlinear displacement-based elements between work points with 4 integration points for each element. The cross-section of each brace was discretized into 128 fibres. The initial out-of-straightness was defined as 0.2% of the effective length of the brace. The potential fracture of braces was considered by implementing the maximum strain range approach proposed by Sen et al. (2019). The gusset plate connection was modelled as discussed in Hsiao et al. (2012). The yield strength of braces and gusset plates had a normal distribution with a mean of 460 MPa and 385 MPa, respectively. A lower limit of 350 MPa was specified for the yield strength of braces, while gusset plates had a lower limit of 300 MPa.

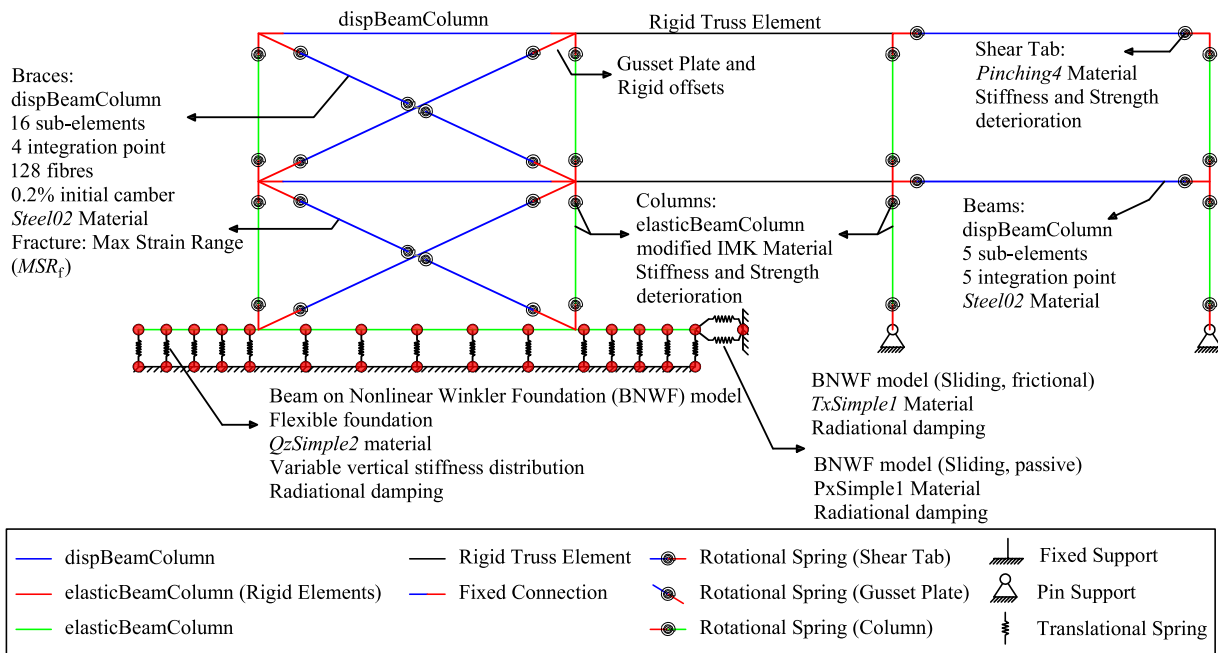


Figure 2. Numerical model of structure-foundation-soil system

Similar to braces, nonlinear displacement-based elements were employed to model the beams in both braced and gravity frame systems. Each beam sub-element had 5 integration points where the Steel02 material model was applied to the fibres of each element. The columns were modelled using a concentrated plasticity approach in which an elastic beam-column element was connected to a spring at each end. The modified Ibarra-Medina-Krawinkler (IMK) deterioration model (Lignos and Krawinkler 2011) was assigned to the column springs, which were capable of capturing the nonlinear behaviour of columns along with strength and stiffness deterioration when applicable. Shear-tab connections in the gravity framing were explicitly modelled to incorporate the effects of connections between beams and columns, following the approach proposed by Elkady and Lignos (2015). A normal distribution with a mean of 375 MPa and a coefficient of variation (COV) of 0.1 was defined for the yield strength of both beams and columns. Additionally, a lower limit of 345 MPa was also specified for beam and column members.

The Beam on Nonlinear Winkler Foundation (BNWF) approach was used to model the shallow foundation and the soil underneath it. By adopting this approach, it was possible to account for various characteristics, including the flexibility of the footing, energy dissipation in the soil, as well as the occurrence of rocking and sliding of the footing (Gajan *et al.* 2008). The foundation was modelled using elastic beam-column elements. On the other hand, zero-length vertical and horizontal springs were employed to account for the soil in contact with the foundation. The QzSimple2 material was applied to the vertical springs to mimic the bearing resistance of the soil. Furthermore, the PxSimple1 and TxSimple1 materials were employed to account for passive sliding and frictional sliding resistance, respectively. Here, the soil strength and stiffness values were determined using the recognized equations in geotechnical engineering (Canadian Geotechnical Society 2006; NIST 2012).

The soil properties were considered as random variables in this study. The properties of the stiff soil and the soft soil representative of site class D were specifically chosen to cover all site class D soil in regard to shear wave velocity. The site with stiff soil had a shear wave velocity (V_{s30}) value between 270 m/s and 360 m/s, while the site with soft soil had a V_{s30} between 180 m/s and 270 m/s. The initial shear modulus was directly calculated using the V_{s30} values mentioned earlier. The variability of the concrete compressive strength of the foundation was also accounted for. The ranges for all random variables considered in modelling the soil and foundation are listed in Table 3. All random variables listed in Table 3 are assumed to have a truncated lognormal distribution. It was assumed that there is no correlation between random variables. The dimensions of the foundation were taken as the decision variables. It was assumed that the length, width, and depth of the foundation had a correlation coefficient of 1.0, meaning that when one of these parameters is known, the values of the other two parameters are also determined. The lower and upper limits for these parameters correspond to the dimensions specified for US footing and CP footing, as listed in Table 2. The NCP footing is also listed in this table to demonstrate the accuracy of assuming a perfect positive correlation between these variables.

Table 3. The range of studied soil properties

#	Soil Parameter	Notation	Stiff			Soft		
			Median	LL	UL	Median	LL	UL
1	Friction angle	ϕ (deg)	35	30	42	5	0	10
2	Cohesion	c (kPa)	5	0	10	35	10	50
3	Initial shear modulus	G_0 (MPa)	212	134	238	65	53	119
4	Poisson's ratio	ν	0.35	0.3	0.4	0.4	0.35	0.45
5	Unit weight	γ (kN/m ³)	18	16	22	16	12	18
6	Concrete compressive strength	f'_c (MPa)	32.5	20	50	32.5	20	50

4. Ground motion selection and scaling

In the Vancouver area, where the buildings of this study are located, the primary contributors to seismic hazard are crustal, subduction in-slab, and subduction interface earthquakes. A set of 60 ground motion records was selected to perform the nonlinear response history analysis. The majority of the ground motion set comprised crustal and in-slab ground motions because these dominate the hazard at the short periods of the studied structures, while a smaller number of interface-type ground motions was also included. These ground motion records were selected from the NGA-West2 (Ancheta *et al.* 2014), K-NET, KiK-net (2019), and CESMD databases. These ground motions were scaled to match the design spectrum of Vancouver City Hall. The

ground motion records were selected as proposed by Baker and Lee (2018), with a scaling factor between 0.5 and 4.0. The crustal selected ground motions had a moment magnitude range of M_W 6.5 to 7.6, a fault rupture distance (R_{rup}) of 10 km to 66 km, and an average shear wave velocity for the top 30 m of soil (V_{s30}) of 192 m/s to 359 m/s. The range of moment magnitude (M_W) for the in-slab ground motions was between 6.7 and 7.4, with corresponding epicentral distances ranging from 30 km to 110 km. On the other hand, interface ground motions had an M_W between 8.0 and 9.0 with an epicentral distance of 110 km to 200 km. Figure 3 illustrates the scaled ground motion records of this study along with the target spectrum.

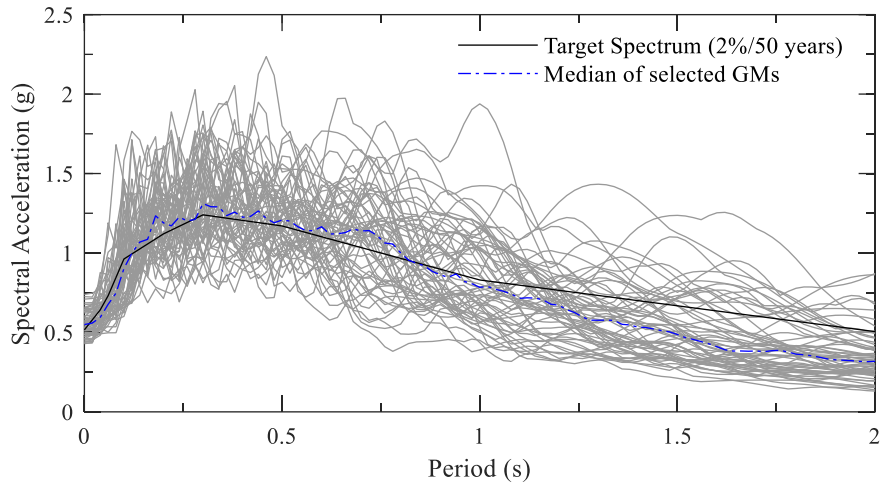


Figure 3. Selected ground motions scaled to design ground motion (2% in 50 years)

5. Results and discussion

The incremental record-wise Latin Hypercube Sampling (LHS) method is utilized to investigate the seismic response of the buildings. This sampling method starts with ten (10) realizations; then the sample size is doubled by adding 10, 20, 40, 80, and 160 new observations, resulting in a total of 320 realizations, where the change in EDPs of studied buildings became minimal by the fifth generation. The subsequent discussion, accompanied by representative plots, is intended to demonstrate the correlation between random variables and selected engineering demand parameters. Each sample data is generated by randomly selecting one value for each variable, depicted by a dot in Figure 4 and Figure 5.

Figure 4 a) shows the correlation between the footing size and the overall inter-storey drift ratio (IDR-GF) of the first storey of the 2-storey building on stiff soil. This figure indicates that as the footing size increases, the overall drift decreases, aligning with the design philosophy of CP footings, where the footing is designed for the capacity of the SFRS, resulting in reduced movement when compared with US footings. Figure 4 b) to Figure 4 d) represent the individual components contributing to the drift shown in Figure 4 a), including the drift due to the braced frame deflection, footing rotation, and footing sliding. These figures demonstrate that as the footing size increases, the contribution of the footing to the overall drift diminishes, while the contribution of the superstructure (IDR-CBF1) increases to the extent that it becomes the predominant factor influencing the overall drift.

The findings of this study indicate that the impact of random variables used to account for uncertainty in gravity loads is insignificant on the maximum IDR of the buildings when assessed individually, as illustrated in Figure 4 e). However, the combined effect of these variables on the IDR and the contributors to the IDR of the studied buildings is found to be significant. Figure 4 f) shows the influence of the variation in axial load on the soil beneath the footing on the first storey IDR of the building on stiff soil, indicating that increasing the axial load reduces the overall drift. Figure 4 g) shows that increasing the axial load on soil also decreases the drifts originating from sliding of the footing. The same observation is made for footing rotation, though the results are not shown here. This observation is consistent with the understanding that as the axial load on the footing increases, it enhances the footing's rotational and frictional sliding resistance.

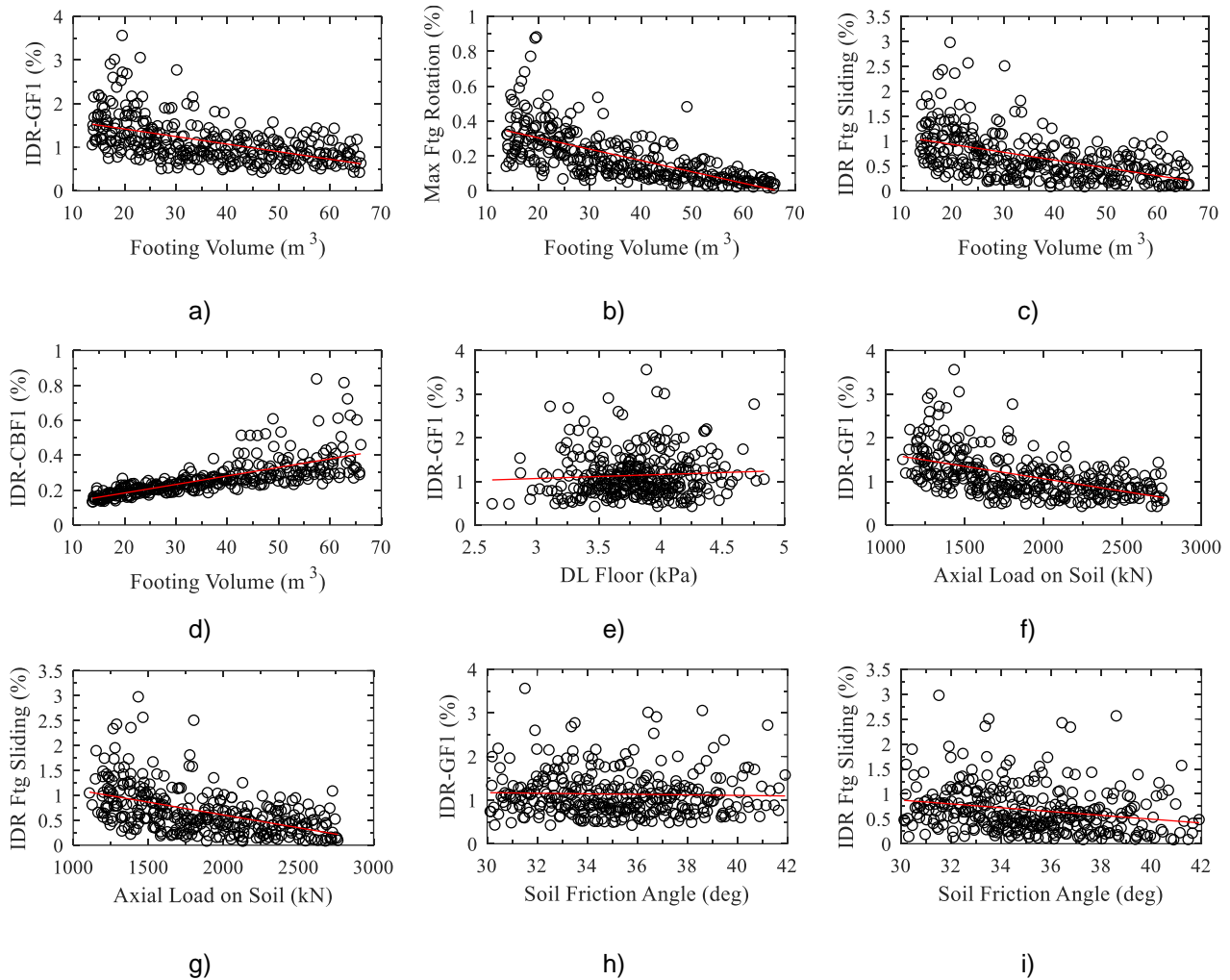


Figure 4. Correlation between selected EDPs and random variables of the 2-storey building on stiff soil: correlation between a) overall first-storey inter-storey drift ratio (IDR-GF1) and footing volume, b) maximum footing (Ftg) rotation and footing volume, c) maximum footing sliding and footing volume, d) first-storey inter-storey drift ratio due to frame deflection (IDR-CBF1) and footing volume, e) floor dead load (DL) and IDR-GF1, f) axial gravity load on soil and IDR-GF1, g) axial gravity load on soil and maximum footing sliding, h) soil friction angle and IDR-GF1, i) soil friction angle and maximum footing sliding

This study also considers the variability in soil properties. Figure 4 h) demonstrates that, for buildings on stiff soil, the overall IDR remains unaffected by changes in the friction angle. However, higher friction angle values correspond to smaller contributions from the foundation to the overall drift, as depicted in Figure 4 i) for IDR due to footing sliding. This figure suggests that for lower friction angles, footing sliding is the primary contributor to overall drift, while for upper friction angles, sliding contributes to less than half as much deformation. This highlights the significance of accounting for variability in soil properties, as alterations in these variables can result in changes in the mode of response. Other results not shown here confirm a similar meaningful impact on EDPs from other soil properties.

As discussed earlier, the size of the footing is considered as a decision variable. Therefore, the impact of the footing size on the overall behaviour of the buildings is investigated in this study. Figure 5 demonstrates the effects of changing the size of the footing on the IDR and peak floor acceleration (PFA) of the 2-storey buildings on stiff soil. Increasing the footing size generally results in higher PFA but lower IDR, whereas decreasing the footing size has the opposite effect, with lower PFA and higher IDR. In order to find an efficient foundation for buildings, the consideration of both footing cost and seismic economic loss is crucial. The seismic economic loss depends on the drift and acceleration that the building experiences during a seismic event. As the size of the footing has opposite effects on IDR and PFA, it becomes pertinent to explore if there exists an optimal point where the seismic economic loss is minimized.

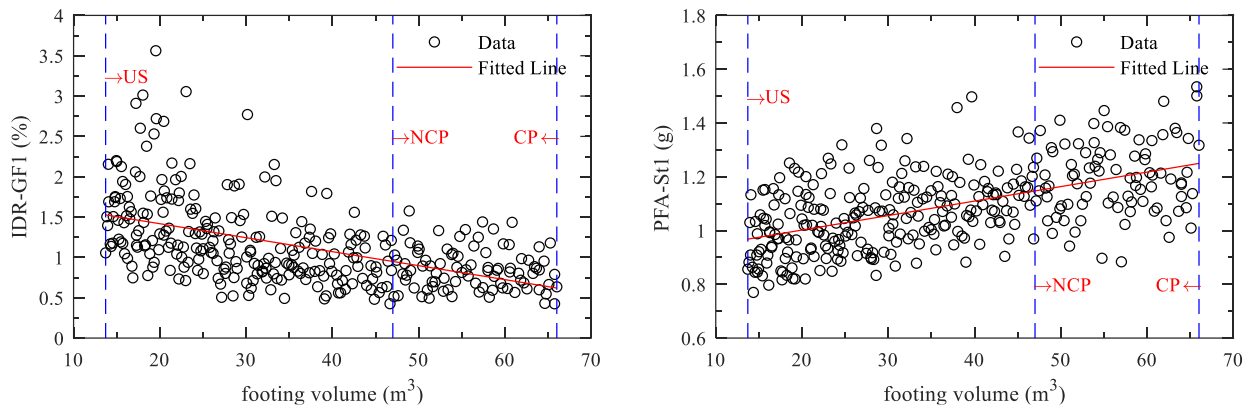


Figure 5. Impact of the footing size on the first-storey IDR and PFA of the 2-storey building on stiff soil

6. Conclusion

This study investigated the performance of Canadian low-rise CBF buildings considering the variability in material properties, gravity loads, and seismic demand on the structure simultaneously, using incremental record-wise Latin Hypercube Sampling (LHS). The results of this study indicate that a change in the size of the footing has contrasting effects on maximum inter-storey drift ratio (IDR) and peak floor acceleration (PFA). This suggests a possibility of finding an optimal size for the footing that strikes a balance between construction cost and the overall seismic performance of the building. The results also emphasize the importance of accounting for variations in building demand, such as the combined effects of gravity loads. Furthermore, random variables such as footing size and soil friction angle have the potential to alter the building's mode of response. Hence, it is essential to account for the impacts of these random variables on the engineering demand parameters (EDPs) to ensure a comprehensive and reliable assessment of the building's performance.

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