

BEHAVIOR OF PENDULUM DAMPER STRUCTURES UNDER THE EFFECT OF EARTHQUAKE AND HARMONIC LOADS

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Abstract: *Tuned mass dampers (TMDs) with different parameters are available to reduce seismic vibrations in structures. If the tuned mass dampers (TMDs) are designed correctly, they contribute significantly to the behavior of the structure by reducing the vibration of the structure. In this study, a pendulum type tuned mass damper (PTMD), which is a tuned mass damper type, was experimentally investigated under the influence of different ground motions. Harmonic and shaking table experiments on the reduced three-storey model, which represents a real three-storey building, were carried out under the influence of three different earthquakes. The harmonic loads used in this research, on the other hand, have a constant amplitude and frequency in the resonance state of the structure. The other three earthquake ground motions, El Centro, Kobe, and Northridge earthquakes, were reduced to the same scale, and the frame system was affected by a uniaxial shaking table. As practiced in this study, PTMDs are usually placed on the top floor of the building. By changing the pendulum length and weights, which are the design parameters of PTMD, its behavior under both harmonic and different real earthquake ground motions is investigated. The tests of the pendulum and pendulum-free system were repeated, with pendulum lengths in the range of 40 mm - 200 mm, and the mass of the pendulum being 1%, 3%, and 5% of the total mass of the structure, respectively. The acceleration and displacement values at the reference floors of the model are measured under these four ground motion conditions. Considering the measurement results, pendulum and pendulumless systems with different parameters were compared. It has been observed that PTMDs under harmonic load significantly decrease the top floor acceleration and each floor displacement values compared to the pendulumless system. Although PTMDs under the effect of earthquake ground motion did not give as good results as harmonic loading compared to the pendulumless system, they decreased the acceleration and displacement values. The characteristics of the earthquake also affect the behavior of the pendulum. When the acceleration and displacement values of the structure under the effect of harmonic loads are examined, it is revealed that it performs better than the real earthquake loads.*

Keywords: *Pendulum Dampers, Vibration Experiments, Tuned Mass Dampers, Harmonic Loads, Earthquakes, Shake Table Test, Model Scaling*

1. Introduction

Buildings are exposed to many dynamic effects such as earthquakes, wind, water waves, and construction equipment. What is required from the structure is that it be safe, durable, and able to adapt itself to provide a

certain level of comfort against these loads. Control systems that can provide this adaptation can be classified as passive, active, semi-active, or base isolation systems. The concept of a tuned mass damper was first put forward by Hermann Frahm in 1909 to eliminate the danger of vibration created by ship machinery on the ship's keel. Frahm named this system, which he patented in 1911, as a tuned vibration damper. This vibration control device introduced by Frahm had no natural damping. The instrument was effective only when the natural frequency of the tuned mass and the external impact frequency were very close to each other (Den Hartog, 1985; Rana and Soong, 1998).

Cuvalci and Ertas (1996) used a simple pendulum mounted on the end mass of a beam subjected to sinusoidal excitation as a damper. The dynamic response of this system has been investigated experimentally and numerically. Nonlinear equations of motion were developed to investigate the autoparametric interaction between the first two modes of the system. The resonance condition occurred when the frequency of the beam was twice the frequency of the pendulum. Experimental results were compared with theoretical results, and they were found to be compatible. This research has shown that a pendulum can be used as a vibration absorber for flexible structures. PTMDs are also used in different areas. Setareh *et al.* (2004) investigated the use of a pendulum-regulated mass damper (PTMD) to control excessive vibrations on building floors. They conducted an analytical study of this system. An equivalent single degree of freedom (SDOF) model for PTMD is considered. The results show that PTMDs can provide practical vibration control for such applications. Setareh *et al.* (2006) conducted analytical and experimental studies of PTMD to control excessive ground vibrations caused by human movements. An equivalent single degree of freedom model for PTMD is considered, and the equations of motion are derived. The optimum design parameters of PTMD were found using an optimization algorithm. It has been shown that, when properly designed, PTMD can significantly reduce excessive ground vibrations.

Gerges and Vickery (2005) derived the equation of motion for a pendulum-type tuned mass damper (PTMD) single-degree-of-freedom system under dynamic force and principal acceleration stimuli. The system with PTMD was investigated under dynamic loads such as wind and earthquakes. It was seen that the damping was successful under these loads, and the optimum design parameters were determined. Elias and Matsagar (2017) investigated passive TMD systems for the control of structures under wind and earthquake effects using TMD. They described and compared TMDs as passive control systems to protect structures subjected to forces due to wind and earthquake ground motions. They then conducted a detailed literature review of TMD, examining both theoretical and empirical research. Their investigations clearly showed that TMDs have the potential to improve the behavior of structures under the influence of wind and seismic movements. Banerjee and Ghosh (2020) tried to find the optimal design of TMD for structures with moving bases. The performance of the passive TMD system largely depends on the design parameters. The calculation of optimum parameters is an important problem in the design of TMD. It is also important to define the parameters of the structure to which the TMD will be attached. This study focuses on this and aims to obtain optimal TMD parameters within the framework of a genetic algorithm (GA). The proposed systems were compared in terms of optimum adjustment ratio and optimum damping ratio for a certain value of the mass ratio in the linear TMD system under the influence of earthquake ground motion. The control performance achieved by different designs of TMD in the tuned state is found to be similar to each other, although they have some minor differences in the values of the optimum parameters. Natesan and Ramasamy (2023) conducted numerical and experimental studies to reduce the vibration of a structure using a single PTMD. A numerical model for the theoretical analysis of PTMD was built using Matlab Simulink. In this study, it was clearly seen that when the pendulum mass of 250 g was added to the structure, the vibrations and displacements in the structure decreased. Deraemaeker and Soltani (2017) initiated a study to obtain an analytical solution for the optimization of a simple PTMD connected to a main system considered as undamped and reduced to a single degree of freedom model. They applied Den Hartog's equal peak method to derive the optimum design. The optimum length of the pendulum and the optimum damping were calculated with the mass ratio between the primary system and the PTMD. The formulas are presented in both dimensional and non-dimensional forms and are easy to implement in real-world applications. Moreover, it is shown that the vibration reduction performance of the proposed optimal design exactly matches the results obtained by numerical optimization techniques. Viet and Nghi (2014) used a nonlinear single-mass, two-frequency pendulum mass damper to reduce horizontal vibration. In a horizontal excitation effect, the single TMD mass has two movements (swing and translation) simultaneously. This model has two

natural frequencies. A numerical simulation was performed to verify the approximate analysis. They showed that the proposed pendulum type TMD has good performance for large vibrations.

Roffel *et al.* (2013) investigated the performance of pendulum regulated mass dampers to reduce the response of flexible structures. The dynamic responses of the flexible multi-degrees-of-freedom structure combined with a three-dimensional movable PTMD were investigated. Roffel and Narasimhan (2014) used PTMDs in full-scale applications to reduce excessive structural movements due to wind. They have optimized the design of control systems such as PTMD. They proposed parameter estimation using the Extended Kalman Filter (EKF). Thus, they were able to predict the dynamic properties of the plain structure. Experimental studies and numerical simulations are presented. The obtained results have been evaluated and shown to provide reliable estimates for the modal parameters required. Roffel and Narasimhan (2016) conducted a full-scale study on the status assessment of PTMD. Frequency and damping adjustment ratios are considered as PTMD design parameters. These parameters were estimated using an extended Kalman filter with the PTMD in service.

Oliviera *et al.* (2014) determined the design criteria for a pendulum damper to control the vibrations of high-rise buildings. Damping was done by adjusting the frequency of the damper to a certain frequency. The aim of this study is to evaluate the efficiency of PTMD in structures subjected to dynamic loads. A set of general dimensionless optimum parameters for PTMD is presented in this research. They designed the PTMD to control any tall building with different mass and damping ratios. Xu *et al.* (2021) investigated the non-linear effects of a single degree-of-freedom system on the vibration control of PTMDs under the harmonic load effect. The nonlinear motion equations of the linear SDOF structure with PTMD are formulated. The results obtained by the harmonic balance method under vibrations of different amplitudes are compared with those obtained using the linearized system. It has been shown that the PTMD can be considered to behave linearly when the PTMD rotation angle is below about 9° . The large mass ratios of PTMD have been found to result in reduced pendulum motion. A parametric study was conducted to find the optimum parameters of nonlinear PTMD under vibrations of different amplitudes. In order to validate the proposed PTMD design method, a 61 meter high steel chimney structure exposed to harmonic and random wind loads was investigated.

Shu *et al.* (2017) aimed to make the optimum seismic design with a heavy bucket suspended in the power station building and a pendulum mass damper system. Structural coal buckets inside the structure were used to assess and minimize the seismic responses of the power station. They have developed and not modeled a full-scale computational model of this example. Modeling has been compared with experimental results and shown to be in agreement. They found the optimum design parameters to increase the performance of the system. Shu *et al.* (2019) aimed to evaluate the seismic weakness in the same plant and to design the equivalent PTMD system in the best way possible so as to minimize its damage. In this study, they designed by performing performance-based analysis.

Aydin *et al.* (2022) conducted experimental studies on the use of TMD to control the behavior of structures under harmonic effects. A 3-storey reduced shear frame model was designed, and PTMD was mounted on the top floor of the structure. Harmonic loads were given to the building model fixed on the uniaxial shaking table, with and without a pendulum, and the acceleration values at floor levels under these loads were measured and the results were compared. Experiments were made for two different values of the mass attached to the tip of the PTMD and two different values of the pendulum length. Models with different PTMD designs were compared with each other and with cases without dampers, and the most suitable damper model was determined. According to the acceleration values measured within the scope of the experimental study, it was observed that PTMD improved the behavior of structures under harmonic effects. In subsequent studies, Aydin *et al.* (2015) investigated the effects of PTMDs on the behavior of the structure. In part of this study, stranded PTMDs were examined. The vibration tests of the 3-story shear frame model with reduced dimensions were performed. The model under the influence of harmonic loads with different frequencies was tested on a uniaxial shaking table with and without a pendulum. Firstly, pendulumless experiments were carried out on the 3-storey reduced building model, and structural responses were found. The pendulum mass was chosen to be approximately 3% of the total mass of the structural model. Afterwards, pendulum experiments with the same model were carried out. Experiments show that the pendulums used are highly effective in the resonance state.

According to the literature review, there are many studies on pendulum-type tuned mass dampers. Considering such systems, it is evident that they have been widely applied to high-rise buildings in the last thirty years. Conducted studies are generally theoretical studies; experimental studies are more limited. PTMD's optimum design parameters are discussed in this study since they can both improve the dynamic behavior of the structures and reduce their cost. It has been observed that PTMD models under the influence of harmonic and real earthquake ground motions reduce acceleration and floor displacements. It was investigated how the changes in pendulum length and pendulum mass of models with PTMD affect the behavior of the structure.

2. Experimental study

2.1 Loading procedure

The loads to be effected on the structure were chosen as the harmonic load in the resonance state and three real earthquake records. Selected real earthquake records are scaled according to the laboratory environment with a certain method. The purpose of designing this scaling method is that it is not always possible to carry out physical and infrastructure experiments in real dimensions. Even if it is done, it will be very costly in economic terms. Instead of working on a real scale, it is possible to reach an approximate conclusion by examining the laboratory experiments of the scaled model that represents the real structure in the laboratory. In this study, real earthquake records should be scaled and applied to the model in the laboratory environment. In the current study, real earthquake records were scaled by taking into account the scaling ratios given in Table 1. While scaling in this study, the scaling factor was determined to be $\lambda=10$. While the acceleration values of the selected earthquakes did not change, they were only scaled as time and these values were used (Hokmabadi, 2014).

Table 1. Scaling relationships in terms of the geometric scaling factor (λ) (Hokmabadi, 2014).

Parameter	Scaling relationship
Mass density	1
Force	λ^3
Stiffness	λ^2
Modulus	λ
Acceleration	1
Shear-wave velocity	$\lambda^{1/2}$
Time	$\lambda^{1/2}$
Frequency	$\lambda^{-1/2}$
Length	λ
Stress	λ
Strain	1
EI	λ^5

As in Figure 1 below, the acceleration values have remained the same, but the time values have been scaled according to $\lambda=10$.

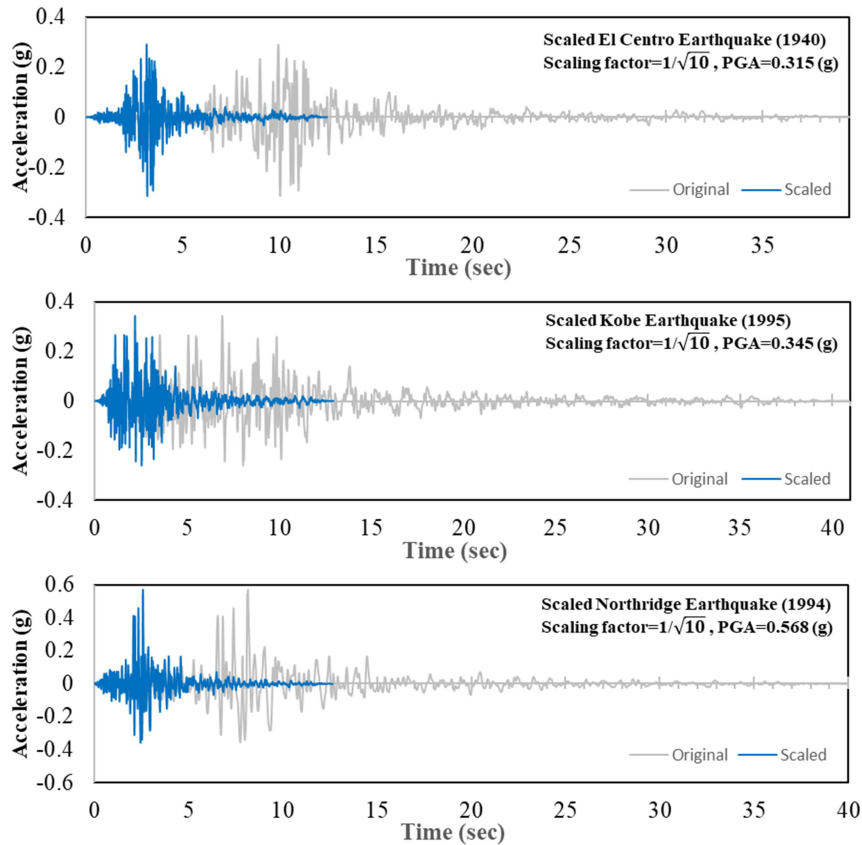


Figure 1 Scaling of real earthquake records.

For the designed models, the harmonic load and real earthquake records with three different properties were used in the shaking table experiments to be performed at laboratory scale. The amplitude of this harmonic load is 5 mm and the frequency is 1.5 Hz, adjusted according to the resonance state of the structures. The real earthquake records were determined by considering the peak surface accelerations, magnitudes and durations. The earthquake records to be used are El Centro (1940), Kobe (1995), and Northridge (1994) earthquake records. The characteristics of these earthquake records are shown in Table 2 below.

Table 2. Earthquakes to be used in the experiments and their properties.

Earthquake	Country	Year	PGA (g)	Magnitude (R)	Duration (s)
El Centro	United States	1940	0.315	6.9	39.49
Kobe	Japan	1995	0.345	6.8	40.90
Northridge	United States	1994	0.568	6.7	39.89

The behavior of the structure in the resonance state corresponding to the first mode is examined. The period in the undamped free vibration of the existing structure and its frequency in the resonance state from the period are found. The harmonic amplitude of the loading in the experiments is 5 mm, the frequency is 1.5 Hz, and the cycle number is 5. No scaling has been made in harmonic loading. It is applied on the system as a time dependent displacement value. Real earthquake records from certain stations are scaled with the above scaling method. These loads were entered into the program as time dependent acceleration and the program affected the model as time dependent displacement. Below are the time dependent graphs of the loadings used in the experiments.

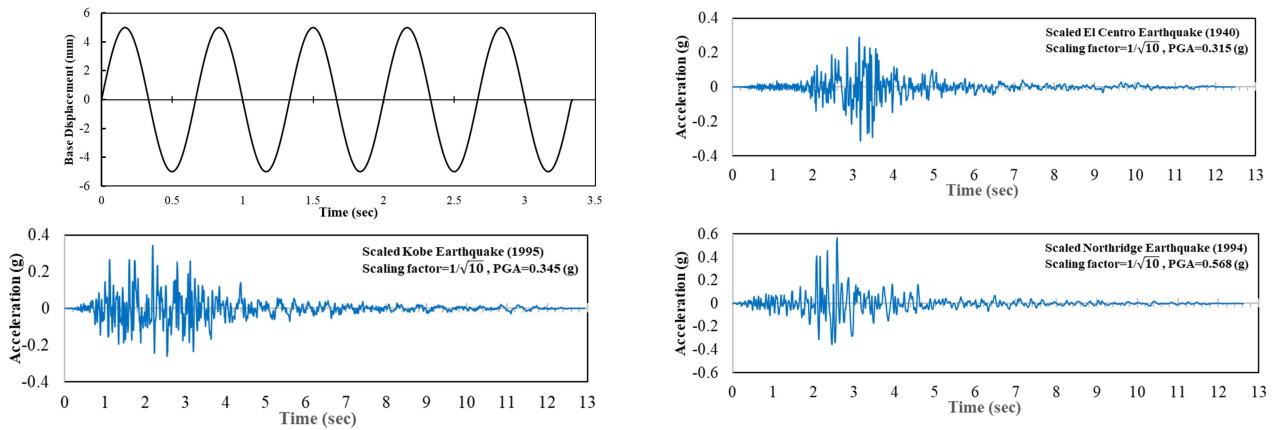


Figure 2 Harmonic and scaled earthquake loads applied to the system.

2.2 The experimental setup

The experimental setup in the structural mechanics laboratory used in this study is shown in detail in Figure 3 below. This assembly consists of a control box and program, a data logger and program, a single axis shaking table, an accelerometer, three linear displacement meters (LVDT) and a small scale three-story steel frame identical to the real structure mounted on the shaking table. The model frame is a uniform structure with a weight of 6 kg and a height of 1000 mm with each floor being 330 mm high, with a modulus of elasticity $E=2 \times 10^5$ MPa and a Poisson’s ratio, $\nu=0.3$. The dimension of the rigid rectangular floor of each floor is 350 mm×305 mm.

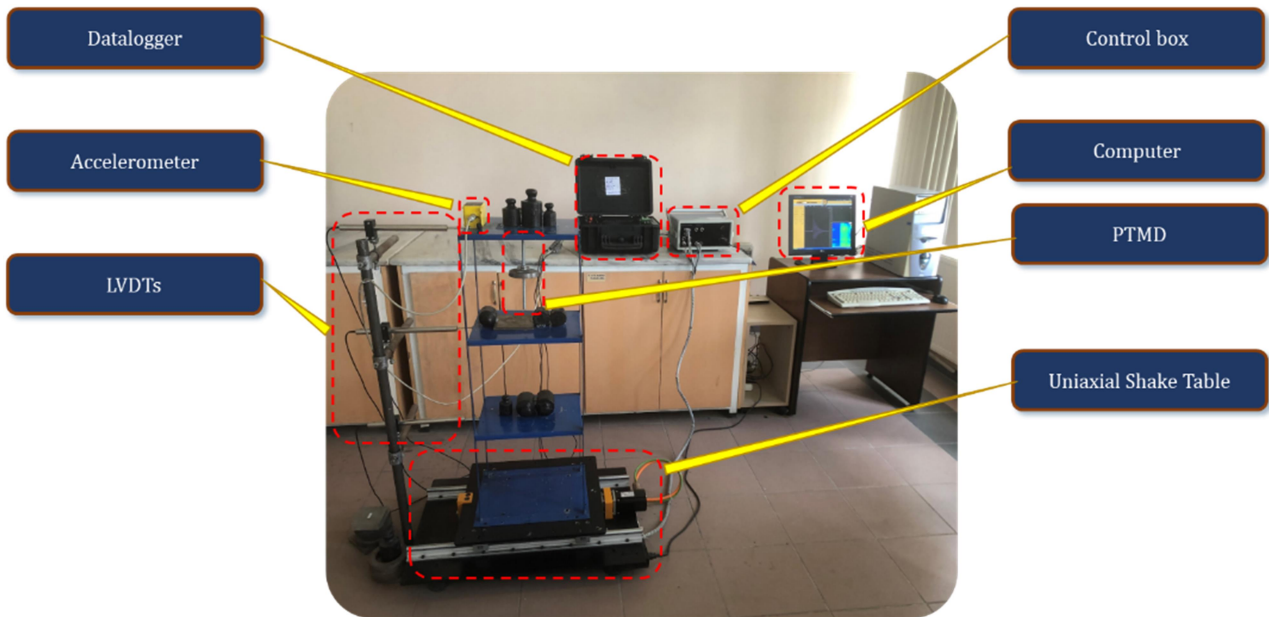


Figure 3: Experimental setup with TMD.

The weights placed on each floor in the model used in this study are the same and are given in the table below. The floors, which consist of a more rigid structure, are supported by four 2.5 mm x 25 mm rectangular columns. To ensure a unidirectional movement, the weak sections of the columns are arranged in the shaking direction.

Table 3: Floor masses of the model.

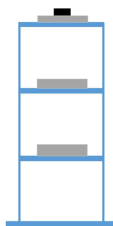
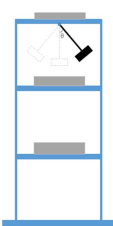
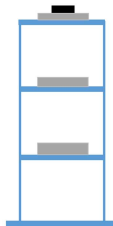
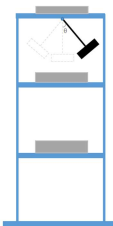
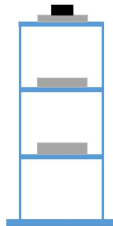
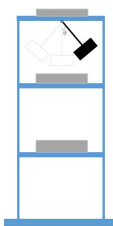
Floor number	Mass from the frame (kg)	Mass placed (kg)	Total floor masses (kg)
1 st	2	4.5	6.5
2 nd	2	4.5	6.5
3 rd	2	4.5	6.5

In this study, the pendulum was placed only on the top floor. Before placing the PTMD, the mass of this pendulum was added as a passive mass to the floor where it would be placed, and the shaking table test of the simple structure was performed. The effect of the design parameters of the pendulum on the behavior of the structure under the influence of applied loads was investigated. LVDTs are installed on each floor, and accelerometers are installed on the top floor.

2.3 Introduction of the models used

The total mass of the structure is the same in all models. Only the length and mass of the pendulum have been changed. Pendulum lengths were changed by 10 mm at each step in the range of 40 mm - 200 mm. The experiments of the undamped and damped system were repeated so that the mass of the pendulum was 1 %, 3% and 5% of the total mass of the structure. The masses of the pendulum type TMDs designed for the damped system are added to the top floor where the pendulum is located as a stationary load to form undamped models. The nomenclatures representing the models used in the experiments are defined below. Figures and drawings of undamped and damped models examined in this study are given in detail in Table 4.

Table 4: Models with and without dampers for Floor 3.

Mass Ratio	Without Damper	With Damper
1%		
3%		
5%		

The figures in the first column are undamped models in which the masses of the pendulum are passively added to the top floor, while the figures in the second column are damped models with a pendulum system. θ represents the oscillation angle of PTMD.

2.4 Evaluation of experiments

While comparing the experimental results obtained within the scope of this study, the values measured by the accelerometer on the top floor of the model and the LVDTs placed on each floor were taken into account. Time dependent acceleration and displacement graphs of the obtained data were also drawn within the scope of this study. Undamped and damped models are compared in detail in the graphics. By looking at the pendulum length and mass of the PTMD system, the model providing the best damping was tried to be obtained. These comparisons were made for each installation within itself.

The accelerometer graph given in the graphs below represents the acceleration-time values of the top floor. LVDT graphics, on the other hand, give the displacement-time values of the floor from the floor. The results and explanations of the experiments under the influence of four different ground motions are given below.

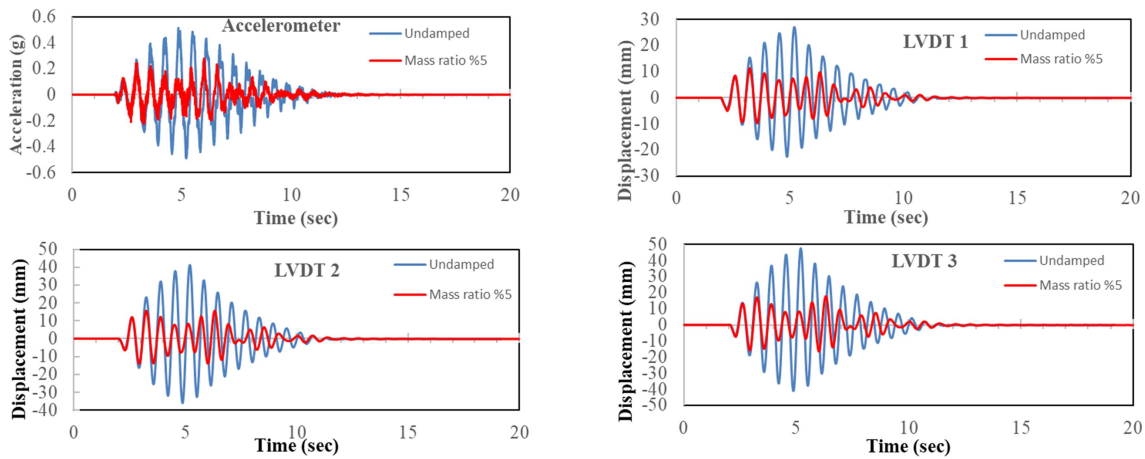


Figure 4: Comparison of acceleration and displacement measurements of models in harmonic loading.

Considering the design parameters of the harmonic pendulum and pendulum system given in Figure 4 above, the test results with the best performance are given. In the systems evaluated by looking at the time dependent acceleration and displacement graphs, it has been observed that the pendulum length is $L=100$ mm and the pendulum/structure mass ratio is 5%. In other experiments under harmonic load, it was observed that the system with PTMD reduced the acceleration of the top floor and the displacement values of the floors quite successfully.

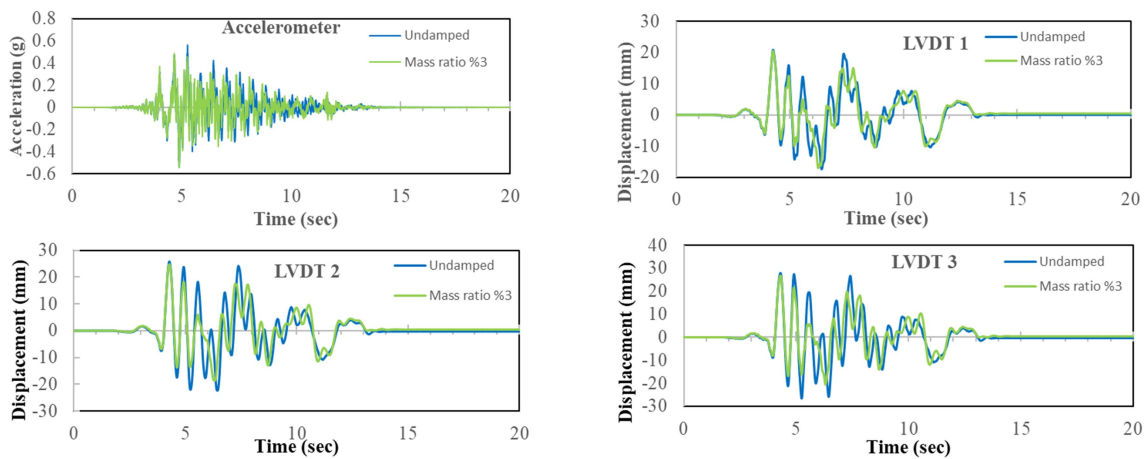


Figure 5: Comparison of the acceleration and displacement measurements of the models in the El Centro earthquake.

The test results with the best performance obtained with and without pendulum under the effect of the El Centro earthquake are given in Figure 5 above. The dynamic behavior of the system was evaluated by looking at the time dependent acceleration and displacement graphs obtained from the experiments. It showed the best performance with pendulum length $L=70$ mm and pendulum/structure mass ratio of 3%. It was observed that the PTMD system successfully reduced the acceleration of the top floor and the displacement values of the floors under the ground motion effect of the El Centro earthquake. Thanks to the PTMD system, the vibration of the top floor was reduced during the process after the peak values of the earthquake. In other experiments, it has been observed that it generally reduces acceleration values and floor displacements. It has been observed that while increasing the top floor acceleration value in some designed pendulums, it decreases the floor displacements.

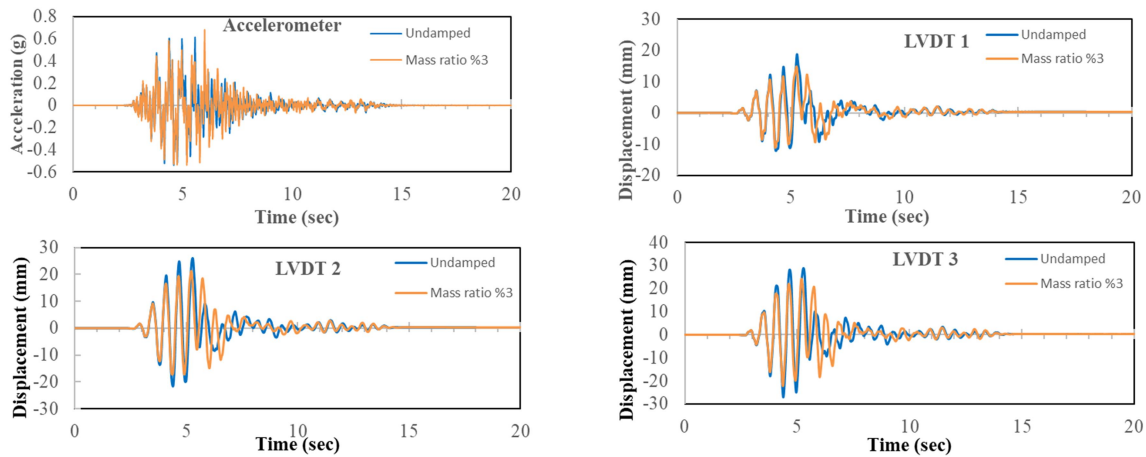


Figure 6: Comparison of the acceleration and displacement measurements of the models in the Kobe earthquake.

The time dependent acceleration and displacement values obtained from the test results with the best performance obtained with and without pendulum under the influence of the Kobe earthquake in Figure 6 above are given. Among the experiments, the best performance was obtained when the pendulum length $L=60$ mm and the pendulum/structure mass ratio was 3%. It was observed that the system with PTMD under the influence of Kobe earthquake ground motion increased the acceleration of the top floor slightly, while it successfully decreased the displacement values of the floors. In other experiments, it was observed that the acceleration values generally increased, but the floor displacements generally decreased.

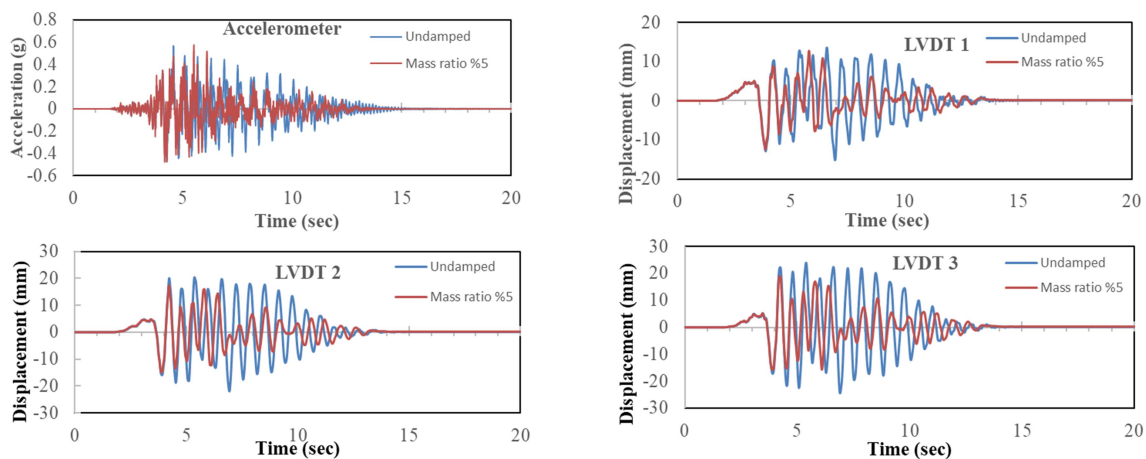


Figure 7: Comparison of models' acceleration and displacement measurements in the Northridge earthquake.

The time dependent acceleration and displacement values obtained from the test results with the best performance obtained with and without pendulum under the influence of the Northridge earthquake, given in Figure 7 above, are given. Among the experiments, the best performance was obtained when the pendulum length was $L=50$ mm and the pendulum/structure mass ratio was 5%. While the PTMD system under the

ground motion effect of the Northridge earthquake decreased the acceleration of the top floor slightly at the beginning, it successfully reduced the oscillation of the top floor after the peak values. It has been observed that each floor reduces the displacement values well. In other experiments, it was observed that the acceleration values generally increased, but the floor displacements generally decreased. In this earthquake effect, especially the floor displacements are quite low.

3. Results and discussion

In this study, it is aimed to investigate the seismic performance of the PTMD added to the structure under dynamic loads such as earthquake and wind effects. For this reason, the dynamic behavior of the 3-storey model, which is subjected to harmonic and 3 different real earthquake ground motions, is examined. By changing the design parameters of pendulum length and pendulum mass ratio, the design parameters giving the best performance for each loading were selected and time dependent graphics were drawn. Considering these results, systems with and without pendulum were compared. In addition, the experimental results of other designed PTMDs are also briefly mentioned. In general, we can say that PTMDs are successful systems. When the acceleration and displacement graphs are examined, the following comments are obtained:

- PTMD exhibits its best performance under the influence of harmonic loads. Both acceleration and floor displacement values have decreased considerably. It has been observed that the harmonic load effect is quite successful compared to the real earthquake ground motion. This is because this load has a single frequency, while loads such as earthquakes have more than one frequency.
- When we look at the top floor acceleration values of the optimally designed model under the real earthquake effect: While there were decreases in the El Centro and Northridge earthquakes, it was seen that the pendulum system increased the acceleration value of the top floor in the Kobe earthquake.
- When we look at the floor displacement values of the optimally designed model under the real earthquake effect: There were good decreases in all the El Centro, Kobe and Northridge earthquakes. The greatest decrease in displacements occurred in the model under the influence of the Northridge earthquake.
- Considering the optimum designs for the real earthquake effect, the acceleration values generally decreased, but it was observed that the displacements of the floors always decreased. Considering the reduction ratios of PTMDs, the reduction in displacements is greater than the reduction in accelerations.
- It has been observed that the pendulum does not perform very well because the earthquake has a very sudden change of direction, and the pendulum does not have enough time to recover itself at peak values in earthquake accelerations. This behavior was most observed in the PTMD under the influence of the Kobe earthquake.
- In other experiments, it was observed that the effectiveness of PTMD increased when the length of the pendulum was increased and the pendulum mass ratio was low.

In this study, it has been shown from the results obtained above that pendulum systems can be an effective damping system when properly designed according to the incoming loads. Although the best performing systems have been taken into account, different variations in accelerations and displacements still remain unknown aspects of these systems. To explain this a little: In the Kobe earthquake, the system increases the acceleration values while decreasing the displacement values. It is planned to investigate the behavior of multiple PTMDs in structures under the effect of real earthquake ground motion, which is planned to be built in the future.

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